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ON A GENERALIZED YORKE CONDITION FOR SCALAR DELAYED POPULATION MODELS

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(Dedicated to Professor István Győri on the occasion of his 60th birthday)

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Abstract. For a scalar delayed differential equation $\dot{x}(t) = f(t, x_t)$, we give sufficient conditions for the global attractivity of its zero solution. Some technical assumptions are imposed to insure boundedness of solutions and attractivity of non-oscillatory solutions. For controlling the behaviour of oscillatory solutions, we require a very general condition of Yorke type, together with a 3/2-condition. The results are particularly interesting when applied to scalar differential equations with delays which have served as models in populations dynamics, and can be written in the general form $\dot{x}(t) = (1 + x(t))F(t, x_t)$. Applications to several models are presented, improving known results in the literature.

1. **Introduction.** Let $C := C([-h, 0]; \mathbb{R})$ be the space of continuous functions from $[-h, 0]$ to $\mathbb{R}$, $h > 0$, equipped with the sup norm $\|\varphi\| = \max_{-h\le\theta\le0} |\varphi(\theta)|$. In the present work, we consider scalar functional differential equations (FDEs)

$$\dot{x}(t) = f(t, x_t), \quad t \ge 0, \tag{1.1}$$

where $f : [0,\infty) \times C \to \mathbb{R}$ is continuous. As usual, x_t denotes the function in C defined by $x_t(\theta) = x(t + \theta), -h \le \theta \le 0$. Clearly, the requirement of f continuous can be weakened (see [5, Chapter 2]); however, existence and continuity of solutions for (1.1) must be assumed.

Our research is mainly motivated by the applications of the so-called 3/2 stability results (see e.g. [6, Section 4.5]) to scalar population models which can be written in the form

$$\dot{x}(t) = (1 + x(t))F(t, x_t), \quad t \ge 0. \tag{1.2}$$

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